Remark on the calculation of the point kinetic component and dynamic response to various perturbations in the moving fuel in a Molten Salt Reactor

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September 4, 2014

CHALMERS

- Task I: calculation of the point-kinetic component in a MSR;
- Task II: calculation of the dynamic space-dependent response in a MSR to various propagating perturbations.

Objectives for calculating the point-kinetic component:

- in the past, due to low computational capacity, most of the calculations in reactor physics were performed in a point-kinetic approximation (easy to calculate);
- easy to interpret the results;
- almost all available measurements made in the past were done in small tightly-coupled cores where the point-kinetics was expected to work fairly well;
- to predict/prevent RIAs (Reactivity Induced Accidents);

Theory: Task I

The dynamical space-dependent behavior of traditional LWR (solid fuel) reactors can be described as:

$$\frac{1}{v}\frac{\partial \phi(x,t)}{\partial t} = D\frac{\partial^2}{\partial x^2}\phi(x,t)+[(1-\beta)\nu\Sigma_f(x,t)-\Sigma_a(x,t)]\phi(x,t)$$

$$+\lambda C(x,t), \tag{1}$$

$$\frac{\partial C(x,t)}{\partial t} = \beta\nu\Sigma_f(x,t)\phi(x,t) - \lambda C(x,t). \tag{2}$$

where $\phi(x,t)$ and $C(x,t)$ stand for the neutron flux and concentration of delayed neutron precursors.

Derivation of the point-kinetic component in LWRs:

- the point-kinetic component can be derived by a special technique consisting of two steps:
  - factorisation of the flux into the amplitude $P(t)$ and shape functions $\psi(x,t)$ (not unique !!!):

$$\phi(x,t) = P(t)\psi(x,t), \tag{3}$$

plus linearization procedure:

$$Q(x,t) = Q_0(x) + \delta Q(x,t) \text{ with } Q = \phi, P, \psi, \tag{4}$$

$$\Rightarrow \delta\phi(x,t) = \underbrace{\delta P(t)\phi_0(x)}_{\text{point-kinetic term}} + \underbrace{P_0\delta\psi(x,t)}_{\text{space-dependent term}} + O^2(\delta), \tag{5}$$

- *i)* derivation of point-kinetic equations (not possible for MSRs):

$$\partial_t \delta P(t) = \frac{\rho_s(t)}{\Lambda_s} - \frac{\overline{\beta}_s}{\Lambda_s}\delta P(t) + \overline{\lambda}_s \delta C(t), \quad (6)$$

$$\Lambda_{d,s}\partial_t \delta C(t) = \frac{\overline{\beta}_s}{\Lambda_s}\delta P(t) + \frac{\overline{\rho}_s(t)}{\Lambda_s} - \overline{\lambda}_s \delta C(t). \quad (7)$$

  ii) projection of the full solution onto the static adjoint flux $\phi_0^\dagger(x)$ (almost always possible);

$$\delta P(t) = \frac{\int_{-a}^{a} \phi_0^\dagger(x)\delta\phi(x,t)dx}{\int_{-a}^{a} \phi_0^\dagger(x)\phi_0(x)dx} \quad (8)$$

Theory: Task I

- decoupling of the equations for the amplitude and shape functions is achieved by the kinetic approximations, which make various assumptions on the shape function;
- in neutron noise theory, which is a linearized theory, the point-kinetic approximation of calculating the amplitude function is “exact”, i.e. it gives the correct result in first order.

The dynamical space-dependent behavior of the fluid fuel reactors (MSR) reactors can be described as:

$$\frac{1}{v}\frac{\partial \phi(x,t)}{\partial t} = D\frac{\partial^2}{\partial x^2}\phi(x,t)+[(1-\beta)\nu\Sigma_f(x,t)-\Sigma_a(x,t)]\phi(x,t),$$

$$+\lambda C(x,t), \tag{9}$$

$$\frac{\partial C(x,t)}{\partial t} + u(x,t)\frac{\partial C(x,t)}{\partial x} = \beta\nu\Sigma_f(x,t)\phi(x,t) - \lambda C(x,t). \tag{10}$$

Theory: Task I

Derivation of the point-kinetic component in MSRs:

- derivation of the point kinetic equations is more complicated and involved;
- compared to LWRs, the traditional factorization technique can not be used to derive the point-kinetic equations since the equations for the amplitude and the shape functions can not be decoupled (the main reason is the presence of additional streaming term which can not be factorized);
- when using linear neutron noise theory, the application of the point-kinetic approximations, gives an incorrect result in first order (due to the different definition of the adjoint for an MSR as to that in a traditional reactor).

Theory: Task I

However, the correct form of the point kinetic component can still be calculated analytically, by an alternative way.

- this is because the full solution can be obtained analytically, and the point kinetic component can be obtained from it by projection.
- on the other hand, it is not possible to derive one single equation, which is not coupled to the shape function equation, and whose solution would yield the correct point kinetic term.

One objective of this talk is to highlight the above points and present the linearly correct form of the point kinetic component.

Calculation of the full solution: Task I

Assuming the following linearization for the time/space-dependent quantities:

$$\Sigma_X(x,t) = \Sigma_{X,0}(x) + \delta\Sigma_X(x,t), \quad (11)$$

$$\phi(x,t) = \phi_0(x) + \delta\phi(x,t), \quad C(x,t) = C_0(x) + \delta C(x,t), \quad (12)$$

after Fourier transform, one gets

$$\begin{pmatrix} \partial_x^2 + B^2(\omega) & \frac{\lambda}{D_0} \\ -\beta\nu\Sigma_f^0 & u_0\partial_x + (\lambda + \imath\omega) \end{pmatrix} \begin{pmatrix} \delta\phi(x,\omega) \\ \delta C(x,\omega) \end{pmatrix} = \begin{pmatrix} S_\phi(x,\omega) \\ S_C(x,\omega) \end{pmatrix}, \quad (13)$$

Eq. (13) can easily be solved with a standard Greens' function technique.

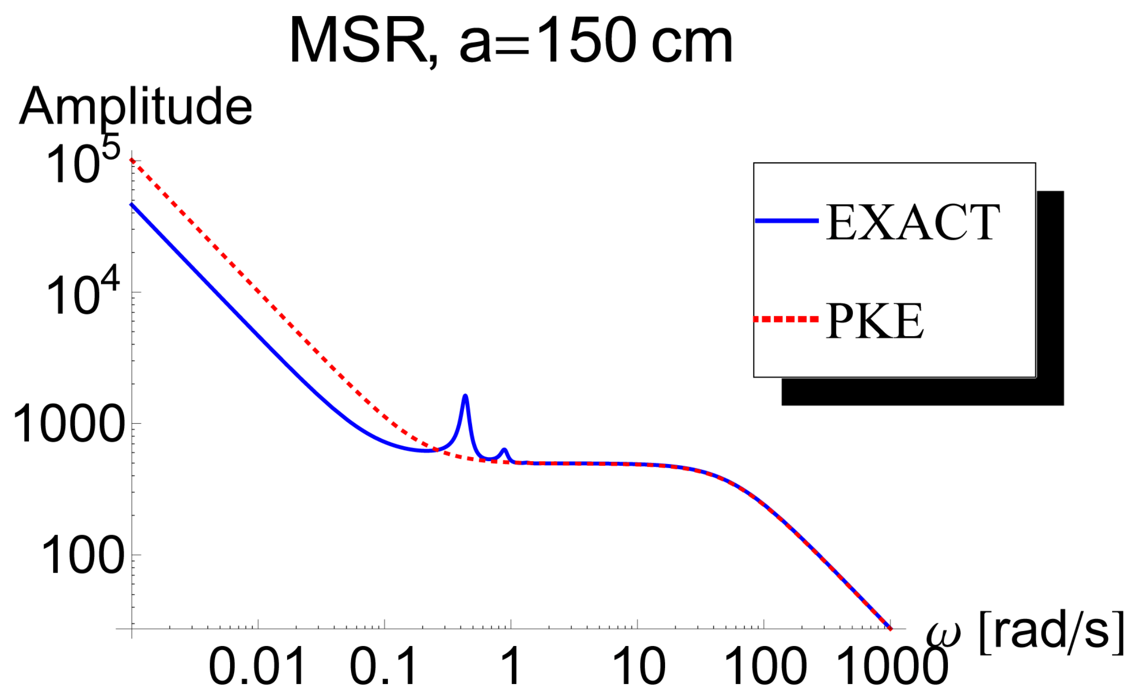

Figure 1: The frequency dependence of the linearly correct solution and that obtained by the point-kinetic approximation.

Conclusions: Task I

- the point-kinetic equations can not be correctly derived in a MSR system using traditional (factorization) approach since the obtained solution does reconstruct the important features of a MSR;
- however, the p.k. component can still be correctly evaluated from the general solution by projection technique.

Objectives: Task II

Objectives for calculating the dynamic space-dependent response induced by propagating perturbations:

- the behavior of the neutron flux in large systems significantly deviates from the point-kinetic component $\Rightarrow$ the space-dependent component is needed to be estimated;
- diagnostic value I: determination of the parameters of different perturbation sources (control rod /fuel assembly vibrations, core barrel vibrations, etc.);
- diagnostic value II: determination of various characteristics of the system (Decay Ratio, coolant velocity, void fraction);
- due to fuel recirculation effect, the dynamical response induced by propagating perturbations in a MSR, might be much stronger as compared to that in LWRs.

Introduction: Task II

In earlier studies only the neutron noise induced by propagating perturbations in absorbtion cross-section was studied. In this work, the other types of propagating perturbations are discussed:

- propagating perturbations in fission cross section;
- propagating perturbations in fission plus absorption cross-sections;
- propagating perturbations in fission/absoption cross-sections and fuel recirculation velocity (have not been studied before!).

As compared to LWRs, due to the recirculation property of the fuel, the structure of the perturbation source in MSRs becomes more complicated and involved which might lead to new effects in the reactor response.

Calculation of the propagation noise: Task II

Following the general procedure in reactor physics, the neutron noise can be calculated as

$$\delta\phi(x,\omega) = \int_{-a}^{a} G(x,x',\omega) S(x',\omega) dx' \tag{14}$$

where $G(x,x',\omega)$ stands for the Greens function (matrix) which can be calculated from Eq. (13) with $S_1(x',\omega)$ and $S_2(x',\omega)$ being replaced by δ-function and $S(x',\omega) = (S_1(x',\omega) \;\; S_2(x',\omega))^T$ is the noise source matrix.

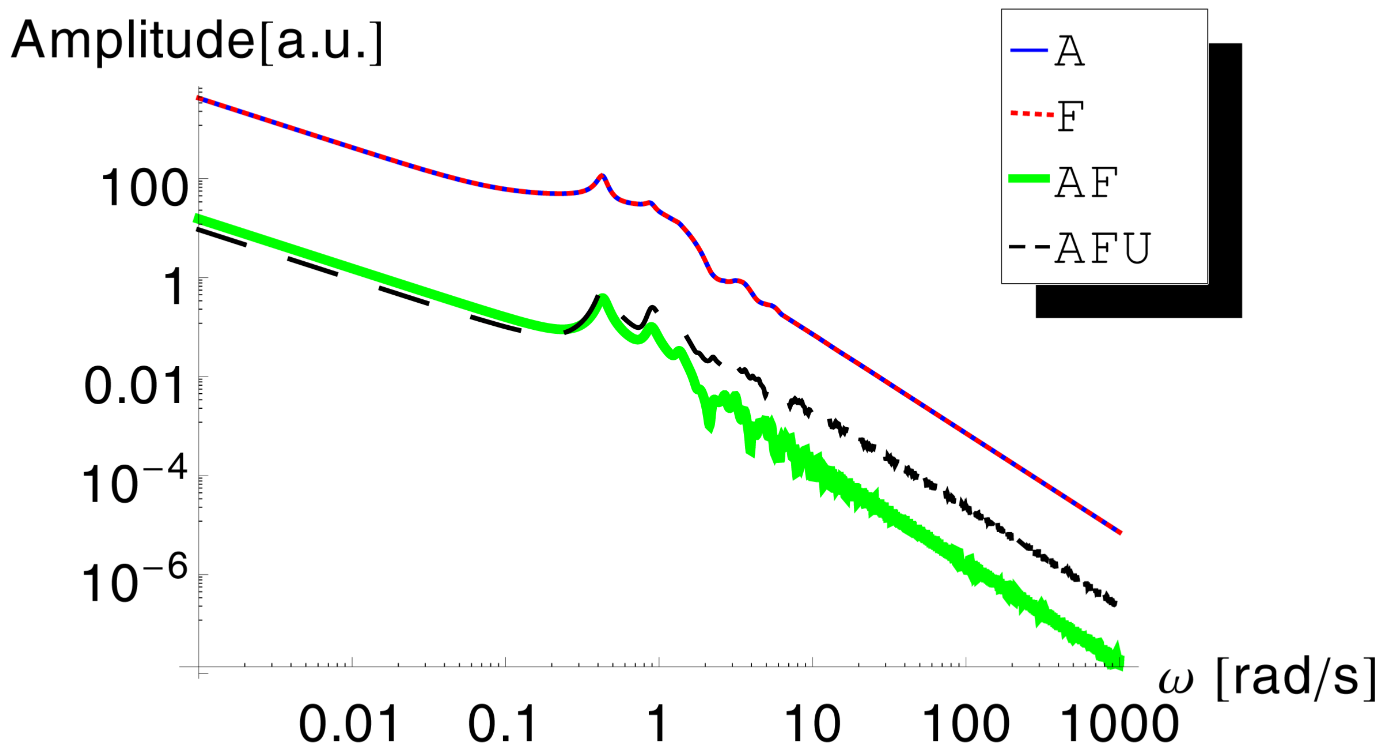

Figure 2: Frequency dependence of the amplitude of the propagation neutron noise for 4 different perturbations for *x*=0 cm (detector position) and $u_0 = 50$ cm/s (propagation velocity).

Conclusions: Task II

- the main finding of the study is that accounting for the fluctuating fuel velocity decreases the total neutron noise level for low frequencies and in opposite increases the latter one for high frequencies as compared to other noise sources.

- another interesting outcome is the observation that the noise induced by propagating perturbations in the absorption and fission cross-sections are quantitatively equivalent despite the fact that the noise structures and calculation procedures differ.

- one should point out that the obtained results and conclusions are valid only for one group theory, however in two group theory the situation can be significantly different.